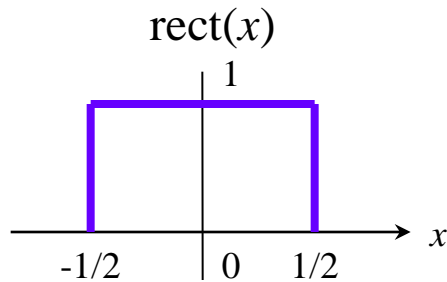


# **Some Fourier Transform Pairs**

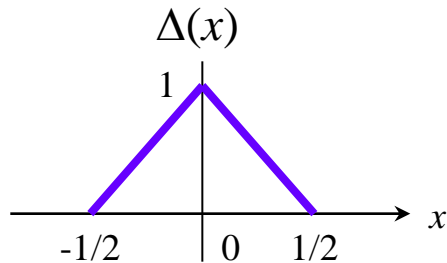
# Useful Functions

- **Unit gate function (a.k.a. unit pulse function)**



$$\text{rect}(x) = \begin{cases} 0 & |x| > \frac{1}{2} \\ \frac{1}{2} & |x| = \frac{1}{2} \\ 1 & |x| < \frac{1}{2} \end{cases}$$

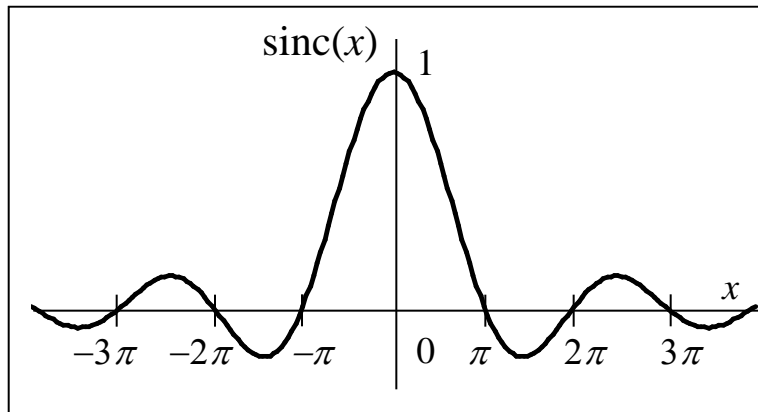
- **What does  $\text{rect}(x / a)$  look like?**
- **Unit triangle function**



$$\Delta(x) = \begin{cases} 0 & |x| > \frac{1}{2} \\ 1 - 2|x| & |x| < \frac{1}{2} \end{cases}$$

# Useful Functions

- **Sinc function**



$$\text{sinc}(x) = \frac{\sin(x)}{x}$$

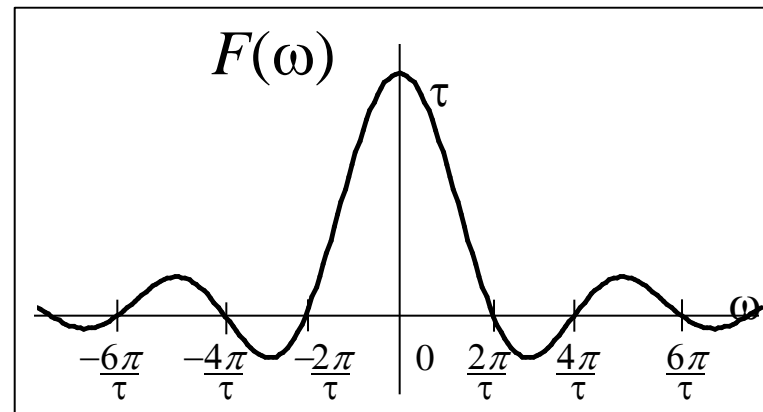
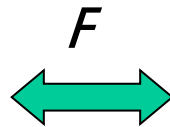
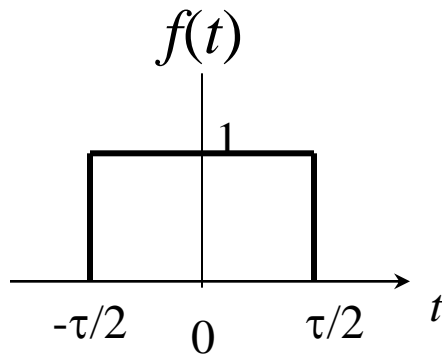
How to compute  $\text{sinc}(0)$ ?

As  $x \rightarrow 0$ , numerator and denominator are both going to 0. How to handle it?

- Even function
- Zero crossings at  $x = \pm\pi, \pm 2\pi, \pm 3\pi, \dots$
- Amplitude decreases proportionally to  $1/x$

# Fourier Transform Pairs

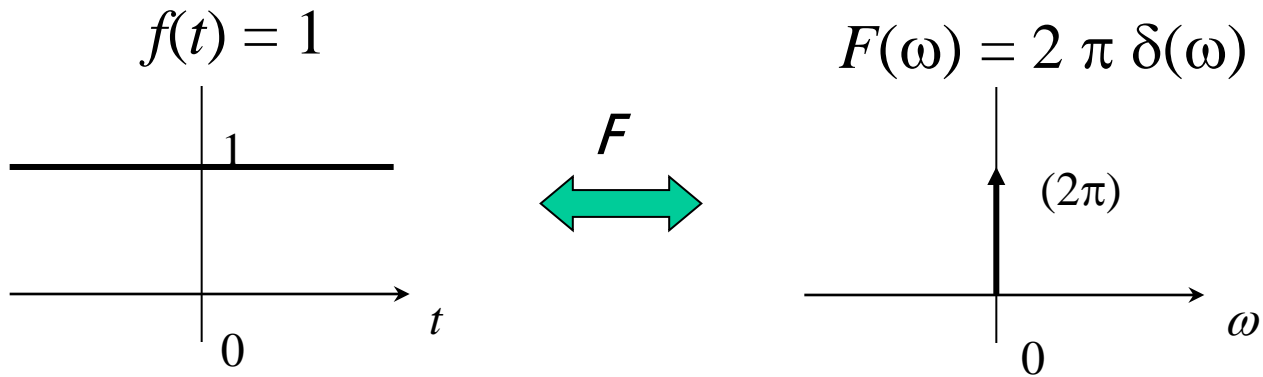
$$F(\omega) = \int_{-\infty}^{\infty} \text{rect}\left(\frac{t}{\tau}\right) e^{-j\omega t} dt = \int_{-\tau/2}^{\tau/2} e^{-j\omega t} dt$$
$$= -\frac{1}{j\omega} \left( e^{-j\omega \tau/2} - e^{j\omega \tau/2} \right) = \frac{2 \sin\left(\frac{\omega\tau}{2}\right)}{\omega} = \tau \frac{\sin\left(\frac{\omega\tau}{2}\right)}{\left(\frac{\omega\tau}{2}\right)} = \tau \text{sinc}\left(\frac{\omega\tau}{2}\right)$$



# Fourier Transform Pairs

From the sampling property of the impulse,

$$F\{\delta(t)\} = \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} dt = e^{-j0t} = 1$$



$(2\pi)$  means that the area under the spike is  $(2\pi)$

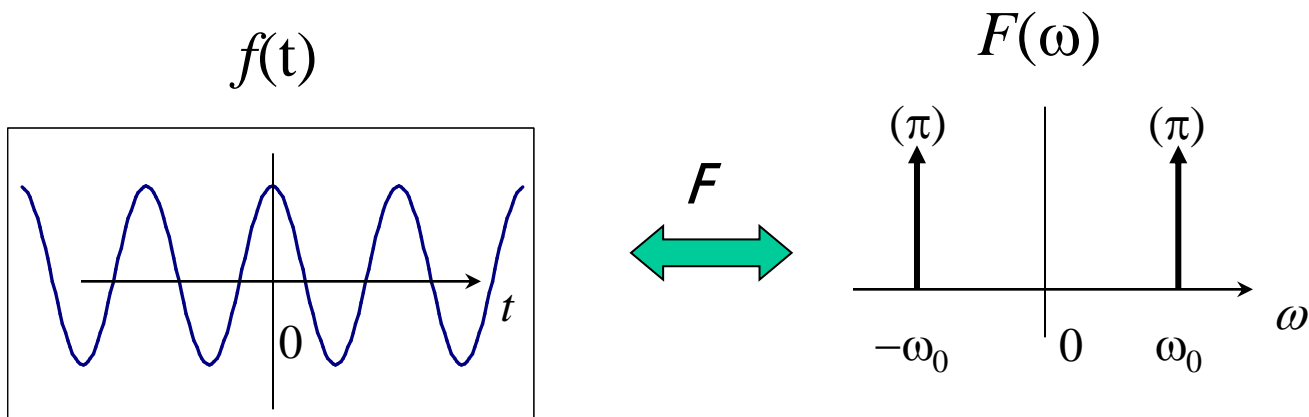
# Fourier Transform Pairs

$$F^{-1}\{\delta(\omega - \omega_0)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \delta(\omega - \omega_0) e^{j\omega t} d\omega = \frac{1}{2\pi} e^{j\omega_0 t}$$

$$\frac{1}{2\pi} e^{j\omega_0 t} \Leftrightarrow \delta(\omega - \omega_0) \quad \text{or} \quad e^{j\omega_0 t} \Leftrightarrow \delta(\omega - \omega_0)$$

$$\text{Since } \cos(\omega_0 t) = \frac{1}{2} (e^{j\omega_0 t} + e^{-j\omega_0 t})$$

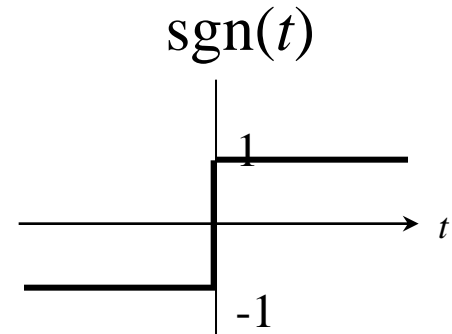
$$\cos(\omega_0 t) \Leftrightarrow \pi [\delta(\omega + \omega_0) + \delta(\omega - \omega_0)]$$



# Fourier Transform Pairs

$$\text{sgn}(t) = \begin{cases} 1 & t > 0 \\ -1 & t < 0 \end{cases} = \lim_{a \rightarrow 0} [e^{-at}u(t) - e^{at}u(-t)]$$

$$\begin{aligned} F\{\text{sgn}(t)\} &= \lim_{a \rightarrow 0} [F[e^{-at}u(t)] - F[e^{at}u(-t)]] \\ &= \lim_{a \rightarrow 0} \left[ \frac{1}{a + j\omega} - \frac{1}{a - j\omega} \right] \\ &= \lim_{a \rightarrow 0} \left[ \frac{-2j\omega}{a^2 + \omega^2} \right] = \frac{2}{j\omega} \end{aligned}$$



# Comparison with Laplace Transform



# Fourier vs. Laplace Transform Pairs

$f(t)$	$F(s)$	Region of Convergence	$F(\omega)$
$e^{-at} u(t)$	$\frac{1}{s + a}$	$\text{Re}\{s\} > -\text{Re}\{a\}$	$\frac{1}{j\omega + a}$
$e^{-a t }$	$\frac{2a}{a^2 - s^2}$	$-\text{Re}\{a\} < \text{Re}\{s\} < \text{Re}\{a\}$	$\frac{2a}{\omega^2 + a^2}$
$\delta(t)$	1	complex plane	1
1	$2\pi\delta(s)$	complex plane	$2\pi\delta(\omega)$
$u(t)$	$\frac{1}{s}$	$\text{Re}\{s\} > 0$	$\pi\delta(\omega) + 1/(j\omega)$
$\cos(\omega_0 t)$			$\pi[\delta(\omega + \omega_0) + \delta(\omega - \omega_0)]$
$\sin(\omega_0 t)$			$j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$
$e^{at} u(t)$	$\frac{1}{s - a}$	$\text{Re}\{s\} > \text{Re}\{a\}$	

Assuming that  $\text{Re}\{a\} > 0$